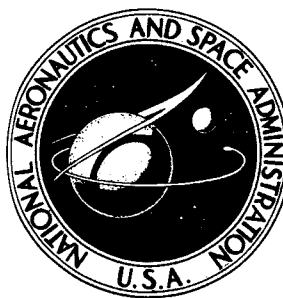


NASA TECHNICAL  
MEMORANDUM



NASA TM X-1581

NASA TM X-1581

GPO PRICE \$ \_\_\_\_\_

CFSTI PRICE(S) \$ \_\_\_\_\_

Hard copy (HC) 366

Microfiche (MF) 66

ff 653 July 65

Facility Form 602

(ACCESSION NUMBER)	10
(PAGES)	1
(NASA CR OR TMX OR AD NUMBER)	
(THRU)	
(CODE)	28
(CATEGORY)	

# ANALYSIS OF FAN-TURBINE EFFICIENCY CHARACTERISTICS IN TERMS OF SIZE AND STAGE NUMBER

*by Warner L. Stewart and Arthur J. Glassman*

*Lewis Research Center*

*Cleveland, Ohio*

ANALYSIS OF FAN-TURBINE EFFICIENCY CHARACTERISTICS IN  
TERMS OF SIZE AND STAGE NUMBER

By Warner L. Stewart and Arthur J. Glassman

Lewis Research Center  
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

---

For sale by the Clearinghouse for Federal Scientific and Technical Information  
Springfield, Virginia 22151 - CFSTI price \$3.00

## ABSTRACT

This report presents a study of fan-drive turbine geometry and efficiency interrelations both in generalized form in terms of specific speed and specific diameter and for a selected set of representative turbofan engine conditions. Results are presented in terms of turbine efficiency and diameter variations as a function of number of stages for a range of fan speeds. Also shown are the variations in engine specific thrust and specific fuel consumption as functions of fan-turbine efficiency for the selected engine conditions.

STAR Category 28

# ANALYSIS OF FAN-TURBINE EFFICIENCY CHARACTERISTICS IN TERMS OF SIZE AND STAGE NUMBER

by Warner L. Stewart and Arthur J. Glassman

Lewis Research Center

## SUMMARY

This report presents a study of fan-drive turbine geometry and efficiency interrelations both in generalized form and for a selected set of representative turbofan engine conditions. The turbine characteristics are based on a constant mean-section diameter, equal stage work, and the use of symmetrical velocity diagrams, which yield close to maximum stage efficiencies. Results are presented in terms of turbine efficiency and diameter variations as a function of number of stages for a range of fan speeds. Also shown are the variations in engine thermodynamic performance parameters as functions of fan-turbine efficiency for the selected engine conditions.

The generalized results, which graphically relate specific speed and specific diameter to number of stages and efficiency, are presented as a means for rapidly obtaining estimates of fan-turbine characteristics for any specified set of engine conditions. Specific speed can be readily estimated from the application requirements; therefore, specific diameter (from which actual diameter can be obtained) and efficiency can be quickly correlated with number of stages.

The analysis results for the selected engine conditions indicate the nature of the efficiency reductions and diameter variations that can accompany reductions in number of stages and fan-tip speed. Efficiency reductions are small in regions of relatively high efficiency, 0.9 or above, but become more severe with continued decreases in number of stages or fan-tip speed. For a fixed fan-tip speed, turbine diameter decreases to a minimum value and then increases again with a decrease in number of stages. For a given number of stages, there is very little change in diameter with variations in fan-tip speed.

For the selected engine conditions, there was about a 3/4 percent penalty in both specific thrust and specific fuel consumption with each one-point reduction in fan-turbine efficiency. Final selection of the optimum diameter and number of stages for the fan-drive turbine must result from a detailed study of the trade-offs between performance and weight for each application.

## INTRODUCTION

Turbofan engines are currently finding wide-spread use in advanced subsonic aircraft due to the substantially improved specific fuel consumption (SFC) and a potential reduction in noise level as compared with turbojet engines. As the demands for further improvements in SFC and noise level are increased, these engines tend toward higher bypass ratios and lower fan-blade speeds. These requirements result in a reduced rotative speed for the fan and its drive turbine, as well as an increase in turbine specific work. The reduced rotative speed and increased work, in turn, result in either an increase in the size and number of stages required to maintain a given efficiency or a decrease in efficiency if a given size and number of stages are to be maintained. It is, therefore, important to understand the interrelation among the number of turbine stages, turbine size, and efficiency, as such information is required in determining a suitable engine configuration for a given application.

This report presents the results of one such study of turbine geometry and efficiency characteristics both in generalized form and for a selected set of representative engine conditions. The generalized results are presented in terms of specific speed and specific diameter, such that they could be applicable to any engine requirement. Included in the report are (1) a description of the method and assumptions used in the study, (2) the analysis results in terms of efficiency and diameter variations as functions of number of stages for a range of speeds, and (3) the results of example calculations of engine performance parameters as functions of turbine efficiency to illustrate the trade-off considerations involved in the final selection of turbine geometry.

## SYMBOLS

D	turbine-exit tip diameter, in. (m)
$D_f$	fan tip diameter, in. (m)
$D_s$	specific diameter, $\text{sec}^{1/2}/\text{ft}^{1/4}$ (dimensionless)
g	gravitational constant, $32.2 \text{ ft/sec}^2$ (1)
$\Delta h$	specific work, Btu/lb (J/kg)
J	mechanical equivalent of heat, 778 ft-lb/Btu (1)
K	loss coefficient, defined in ref. 3
N	rotative speed, rev/min (rad/sec)
$N_s$	specific speed, $\text{ft}^{3/4}/(\text{min})(\text{sec}^{1/2})$ (dimensionless)

n	number of stages
Q	volumetric flow at turbine exit, ft <sup>3</sup> /sec (m <sup>3</sup> /sec)
R	ratio of hub radius to tip radius at turbine exit
Re	Reynolds number, defined in ref. 3
U	turbine mean-section blade speed, ft/sec (m/sec)
U <sub>f</sub>	fan tip speed, ft/sec (m/sec)
V	absolute velocity, ft/sec (m/sec)
W	relative velocity, ft/sec (m/sec)
$\alpha$	stator angle measured from axial direction
$\eta$	total efficiency
$\lambda$	turbine speed-work parameter

Subscripts:

a	first stage
i	intermediate or last stage
id	ideal
u	tangential component
x	axial component
0	stator inlet
1	stator exit
2	rotor exit

Superscript:

—	overall
---	---------

## ANALYSIS METHODS AND ASSUMPTIONS

As indicated in the INTRODUCTION, the fan-drive turbine to be studied herein is of the multistage type. A schematic drawing of a turbofan engine with the various components indicated is presented in figure 1. The fan-drive turbine mean-section diameter is assumed to be constant, and the stage velocity diagrams (except for the first-stage stator) are assumed to be the same. A detailed design for a particular application would undoubtedly involve some tailoring of work, diameter, swirl, and flow angle distributions

among the stages, but this should not affect the significant results of this analysis. The efficiency is based on total- to total-conditions across the turbine.

## Velocity Diagrams

A study of stage diagram characteristics as related to expected efficiency was made in reference 1. This reference study used a speed-work parameter  $\lambda$  as the principal term relating efficiency to the tangential components of the velocities. By definition, for a stage,

$$\lambda = \frac{U^2}{gJ \Delta h} = \frac{U}{\Delta V_u} \quad (1)$$

For a given value of  $\lambda$ , the swirl split between the stator exit and rotor exit was indicated to be different depending on whether static or total efficiency is optimized. When maximum total efficiency was desired, rather large stage exit swirls were indicated, with the rotor reaction approaching that of the stator. In view of these results, the symmetrical velocity diagrams, where the relative velocities leaving the rotor are specified to be equal to the absolute velocities leaving the stator, were selected for this study. This type of diagram is illustrated in figure 2, where station and velocity nomenclature are indicated.

The tangential components of velocity, therefore, can be equated as

$$\frac{V_{u,1}}{\Delta V_u} = - \frac{W_{u,2}}{\Delta V_u} \quad (2a)$$

and

$$\frac{V_{u,2}}{\Delta V_u} = - \frac{W_{u,1}}{\Delta V_u} \quad (2b)$$

Combining equations (1) and (2) with

$$\frac{V_{u,1}}{\Delta V_u} - \frac{V_{u,2}}{\Delta V_u} = 1 \quad (3)$$

the tangential components of absolute velocity can be written in terms of  $\lambda$ .

$$\frac{V_{u,1}}{\Delta V_u} = \frac{\lambda + 1}{2} \quad (4a)$$

$$\frac{V_{u,2}}{\Delta V_u} = \frac{\lambda - 1}{2} \quad (4b)$$

For the symmetrical velocity diagrams selected,

$$V_{x,2} = V_{x,1} = V_x \quad (5)$$

The axial velocity can be expressed in terms of  $\lambda$  and stator-exit flow angle  $\alpha_1$  as follows.

$$\frac{V_x}{\Delta V_u} = \left( \frac{V_{u,1}}{\Delta V_u} \right) \left( \frac{V_x}{V_{u,1}} \right) \quad (6)$$

From equation (4a) and the velocity diagram geometry, therefore,

$$\frac{V_x}{\Delta V_u} = \left( \frac{\lambda + 1}{2} \right) \cot \alpha_1 \quad (7)$$

The selection of stator-exit angle was made with the aid of reference 2. From the above equations and reference 2, it can be shown that (1) for constant  $\lambda$  values, total efficiency is maximized at an angle of  $60^\circ$  to  $65^\circ$  (which is in agreement with the approximately  $60^\circ$  indicated in ref. 3), and (2) for constant flow coefficient ( $V_x/U$ ) values, total efficiency is maximized at an angle of  $50^\circ$  to  $55^\circ$ . A value of  $55^\circ$ , which slightly favors higher through-flow velocities and, thus, smaller diameters, was selected as a base value for this analysis. The effect of varying stator-exit angle will be indicated for one particular case.

It should be emphasized that the use of symmetrical velocity diagrams result in some rather large (about  $35^\circ$  for  $\lambda = 0.4$  and  $45^\circ$  for  $\lambda = 0.2$ ) swirl angles leaving the stage. Since swirl velocities leaving the last stage of a fan-drive turbine represent a loss, this probably would not be a tolerable situation for an actual turbine design. In practice, the last-stage diagram would have to be modified to reduce this exit swirl. For this preliminary type of parametric analysis, such modifications were not considered.



## Stage Efficiency

The general method used to obtain stage total efficiency as a function of the requirements imposed and the type of diagrams utilized is described in reference 3. In using this method for symmetrical velocity diagrams, the following is to be noted.

(1) Equations (4a) and (4b) of this report, rather than the equations presented in reference 3, are used to relate the tangential components of velocity to  $\lambda$ .

(2) The term  $f(\lambda)$ , equation (22) of reference 3, was modified to reflect the type of velocity diagram assumed for this study as compared to those used for reference 3. For this study,  $f(\lambda) = 2 - \lambda$  was used for all stators beyond the first.

(3) To obtain a value for the loss-coefficient term  $(KRe^{-1/5})$ , a stage total efficiency of 0.93 was selected at a  $\lambda = 1$  and  $\alpha_1 = 55^\circ$ . This resulted in a value of  $(KRe^{-1/5}) = 0.01774$ , which was used for this study.

The stage total efficiency characteristics as obtained for the prescribed symmetrical velocity diagram conditions are presented in figure 3, where total efficiency is shown as a function of stage speed-work parameter,  $\lambda$ . Two curves are shown, one for the first stage and one for the remaining stages. The total efficiency of the latter stages is less than that of the first stage due to the increased kinetic energy entering the latter stages together with the allowance for increased loading ( $f(\lambda)$  term). It is also seen that the efficiency is rather high over a wide range of  $\lambda$ , with a marked drop-off starting to occur as  $\lambda$  is reduced below about 0.25 to 0.30. As prescribed, at a  $\lambda = 1$  the total efficiency is 0.93.

Once the stage efficiencies are obtained, the overall total efficiency, neglecting the reheat effect, can be computed from

$$\bar{\eta} = \frac{n}{\frac{1}{\eta_a} + \frac{n-1}{\eta_i}} \quad (8)$$

## Geometry Characteristics

The fan-spool rotative speed is determined by the tip diameter and tip speed specified for the fan.

$$N = \frac{720U_F}{\pi D_F} \quad (9)$$

The following equations, which can be used for any constant mean-diameter turbine with symmetrical diagrams, are then used to establish the fan-turbine geometry character-

istics. First, there is the relation between mean-section blade speed and exit tip diameter

$$U = \frac{\pi ND(1 + R)}{1440} \quad (10)$$

Applying the continuity equation at the turbine exit yields

$$V_x = \frac{576Q}{\pi D^2 (1 - R^2)} \quad (11)$$

Since the stage works are equal, equation (1) can be written

$$\lambda = \frac{U^2 n}{gJ \Delta h} \quad (12)$$

Dividing equation (7) by equation (1) yields

$$\frac{V_x}{U} = \left( \frac{\lambda + 1}{2\lambda} \right) \cot \alpha_1 \quad (13)$$

Turbine rotative speed, specific work, and mass flow are determined by the application requirements. With assumed values for turbine stator angle and exit radius ratio, there are four equations (eqs. (10) to (13)) and five unknowns ( $U$ ,  $D$ ,  $V_x$ ,  $\lambda$ , and  $n$ ). Thus, specification of any of these variables, such as  $n$ , yields unique solutions for the other four. Since the value of exit volume flow  $Q$  depends on turbine efficiency, which is a function of  $\lambda$ , it is necessary to iterate to the final solution.

## Similarity Parameters

Determination of the geometry characteristics from equations (10) to (13) requires specific knowledge of the flow, work, and rotative speed for the given application. If the similarity parameters (discussed in ref. 4) overall specific speed,

$$\bar{N}_s = \frac{NQ^{1/2}}{(J \Delta h_{id})^{3/4}} \quad (14)$$

and overall specific diameter,

$$\bar{D}_s = \frac{D (1 + R)(J \bar{\Delta h}_{id})^{1/4}}{24Q^{1/2}} \quad (15)$$

are used to represent rotative speed and diameter, respectively, then the calculations can be generalized. In this manner, the number of stages, specific diameter, specific speed, and efficiency can be interrelated without knowledge of the specific application requirements. This is accomplished by combining equations (14) and (15) with equations (10) to (13) to eliminate  $D$ ,  $N$ ,  $Q$ , and  $\bar{\Delta h}_{id}$ .

## RESULTS OF ANALYSIS

The analysis results are presented in three parts.

(1) Generalized turbine characteristics are presented in terms of specific speed and specific diameter.

(2) The geometry-efficiency interrelations as a function of fan speed, turbine stator angle, and turbine-exit radius ratio are presented for a selected set of engine conditions. The presented turbine characteristics correspond to speed-work parameters,  $\lambda$ , less than or equal to one, a conservative value that yields a relatively high efficiency. The  $\lambda = 1$  limit corresponds to a turbine efficiency of 0.93 except where otherwise indicated.

(3) The effect of fan-turbine efficiency on engine cycle performance is shown.

### Generalized Turbine Characteristics

Overall specific diameter is plotted against overall specific speed in figure 4 with both stage number and efficiency as parameters. This figure is for a stator exit angle of  $55^\circ$  and an exit radius ratio of 0.6. The increases in efficiency and specific diameter with increasing number of stages at constant specific speed and with increasing specific speed at a constant number of stages are generally well established trends requiring no further discussion.

This figure serves as a means for rapidly obtaining estimates of fan-turbine efficiency and geometry characteristics for any specified set of engine conditions. With rotative speed, mass flow, turbine work, and turbine inlet conditions being set by the specified engine and fan requirements, an assumption for turbine efficiency is all that is necessary in order to obtain an estimate of the overall specific speed. For this value of

specific speed, efficiency and specific diameter can be determined as a function of number of stages from figure 4. If the efficiency values read from figure 4 are not sufficiently close to the assumed value, specific speed can be recomputed and the use of figure 4 repeated. Turbine diameter is then computed from the specific diameter, engine requirements, and efficiency. In this way, a set of turbine characteristics for a specific application can be obtained from figure 4 instead of directly from the analysis equations.

## Turbine Characteristics for Example Application

A set of engine conditions representative of a high bypass turbofan application was selected in order to study the efficiency-geometry characteristics of the fan-drive turbine. The selected engine conditions and resultant fan-turbine requirements at the engine design point, which is at a Mach number of 0.82 and an altitude of 35 000 feet, are presented in table I. This engine has a bypass ratio of 5 and an overall pressure ratio of 24.5.

The characteristics of the fan-drive turbine for this application are presented in figure 5, where number of stages is plotted against fan-tip speed with efficiency and diameter ratio (ratio of turbine-exit tip diameter to fan-tip diameter) as parameters. This figure is for a stator exit angle of  $55^{\circ}$  and a turbine-exit radius ratio of 0.6. The decrease in efficiency with a reduction in number of stages is the same trend noted in figure 4. Whereas it was shown in figure 4 that a reduction in number of stages for any constant specific speed resulted in a continual decrease of specific diameter, it can be seen from figure 5 that for a constant fan-tip speed, the actual diameter tends to reach a minimum value and then increase with a further reduction in number of stages. This reversal in trend is due to the rapidly increasing value of exit volume flow as efficiency decreases. At a fan-tip speed of 1000 feet per second (304.8 m/sec), for example, the efficiency and diameter ratio for eight stages are about 0.93 and 0.73, respectively. As the number of stages is reduced to 2, the efficiency is reduced to about 0.83 while the diameter ratio is reduced to about 0.6, which is the minimum value for this fan-tip speed.

The allowable tip speed for the fan is seen to have a major effect on the turbine characteristics. Lower tip speeds require more stages to achieve a given efficiency level or a reduction in efficiency for a given number of stages. For example, at a fan-tip speed of 1400 feet per second (427 m/sec), the use of two or three stages yield efficiencies of about 0.90. A reduction in blade speed to 700 feet per second (213.5 m/sec) results in a requirement for seven stages along with a diameter increase of about 20 percent to achieve the same efficiency level. The original two or three stages would yield efficiencies in the 0.75 to 0.80 range. For the most part, there is very little change in

diameter ratio with changing fan-tip speed for a given number of stages.

The effects of varying turbine-exit radius ratio and stator-exit angle on the turbine characteristics are shown in figures 6 and 7, respectively, for a fan-tip speed of 1000 feet per second (304.8 m/sec). Number of stages is plotted against turbine-exit radius ratio in figure 6 with efficiency and diameter ratio as parameters. For a given number of stages, an increase in radius ratio results in increases in both diameter and efficiency. The increase in diameter is required by the reduction in blade height, while the increase in efficiency results from the increase in blade speed. With four stages, for example, increasing the radius ratio from 0.4 to 0.8 results in efficiency increasing from about 0.85 to 0.92 and diameter ratio increasing from 0.55 to 0.83.

Number of stages is plotted against stator-exit angle in figure 7 with efficiency and diameter ratio as parameters. For a given number of stages, increasing stator-exit angle results in an increase in diameter ratio and a relatively small change in efficiency. The increase in diameter results from the increased area requirement caused by the reduction in axial velocity. With four stages, for example, increasing stator exit angle from  $45^{\circ}$  to  $70^{\circ}$  results in diameter ratio increasing from about 0.55 to 0.85 and efficiency increasing from about 0.87-0.88 to 0.90-0.91.

## Engine Cycle Performance

It has been shown previously that reductions in fan-turbine diameter and number of stages are accompanied by reductions in turbine efficiency. These efficiency reductions, in turn, cause reductions in the performance of the engine cycle. In order to assess the magnitude of these cycle performance penalties, thermodynamic cycle calculations for a range of fan-turbine efficiencies were made for the selected engine conditions that were shown in table I. The computer program of reference 5 was modified for a two-spool engine in order to make these calculations.

In figure 8 is presented specific thrust and specific fuel consumption as functions of fan-turbine efficiency. For the selected engine conditions, a reduction in fan-turbine efficiency from 0.95 to 0.75 results in about a 15 percent reduction in specific thrust and a corresponding increase in specific fuel consumption. On the average, this is about a 3/4 percent penalty for each of these performance parameters with each one point reduction in fan-turbine efficiency. If engine air flow is maintained constant, these penalties will be somewhat offset by the reduction in turbine size and, consequently, in engine weight. If engine air flow must be increased to regain the lost thrust, then the whole engine must grow in size. Selection of optimum size and number of stages for the fan-drive turbine requires a more detailed design study of the engine.

## CONCLUDING REMARKS

This report presented a study of fan-drive turbine geometry and efficiency interrelations both in generalized form and for a selected set of representative turbofan engine conditions. Results are presented in terms of efficiency and diameter variations as functions of number of stages for a range of speeds. Also shown are the variations in engine thermodynamic performance parameters as functions of fan-turbine efficiency for the selected engine conditions.

From the generalized results, which relate specific speed and specific diameter to number of stages and efficiency, estimates of fan-turbine characteristics for any specified set of engine conditions can be made. The results for the selected engine conditions illustrate quantitatively the nature of the efficiency reductions and diameter variations that can accompany reductions in number of stages and fan-tip speed. These example calculations show that limitations placed on the tip speed of the fan in order to reduce noise levels can place severe requirements on the turbine and result in either lower efficiency levels or more stages than previously encountered in jet engine applications. Lower efficiencies reduce engine thermodynamic performance while increases in number of stages and diameter increase engine weight. It is up to the engine designer to evaluate the trade-offs for a given application in order to achieve an optimum design.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, February 29, 1968,  
126-15-02-17-22.

## REFERENCES

1. Stewart, Warner L.: Analytical Investigation of Single-Stage-Turbine Efficiency Characteristics in Terms of Work and Speed Requirements. NACA RM E56G31, 1956.
2. Smith, S. F.: A Simple Correlation of Turbine Efficiency. J. Roy. Aeron. Soc., vol. 69, no. 655, July 1965, pp. 467-470.
3. Stewart, Warner L.: A Study of Axial-Flow Turbine Efficiency Characteristics in Terms of Velocity Diagram Parameters. Paper No. 61-WA-37, ASME, 1961.
4. Glassman, Arthur J.; and Stewart, Warner L.: Use of Similarity Parameters for Examination of Geometry Characteristics of High-Expansion-Ratio Axial-Flow Turbines. NASA TN D-4248, 1967.

5. Vanco, Michael R. : Computer Program for Design-Point Performance of Turbojet and Turbofan Engine Cycles. NASA TM X-1340, 1967.

TABLE I. - ENGINE DESIGN CONDITIONS

Fan:	
Total mass flow, lb/sec (kg/sec)	360 (163.3)
Pressure ratio	1.6 (1.6)
Tip diameter, in. (m)	68 (1.73)
Compressor spool:	
Air mass flow, lb/sec (kg/sec)	60 (27.2)
Compressor pressure ratio	15.3 (15.3)
Turbine inlet temperature, °R (°K)	2210 (1228)
Fan turbine:	
Inlet temperature, °R (°K)	1551 (862)
Inlet pressure, psia (N/cm <sup>2</sup> )	26.5 (18.3)
Gas mass flow, lb/sec (kg/sec)	60.84 (27.6)
Specific work, Btu/lb (J/kg)	101.4 (2.36×10 <sup>5</sup> )

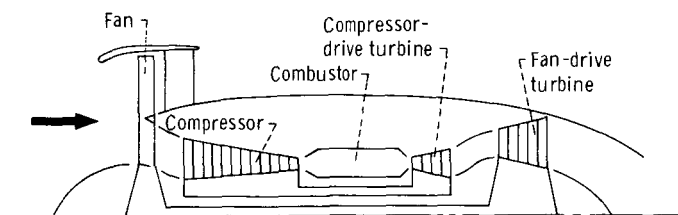
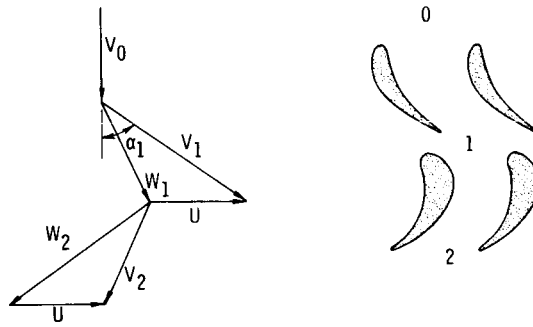
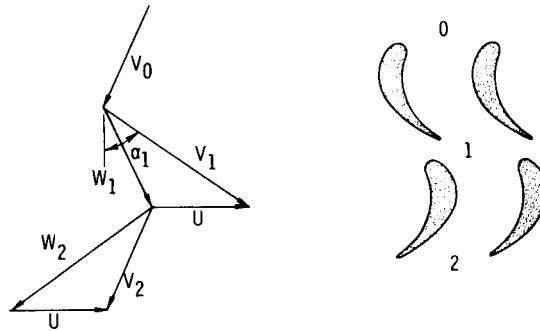


Figure 1. - Fan engine.





(a) First stage.



(b) Intermediate or last stage.

Figure 2. - Stage velocity diagrams and nomenclature.

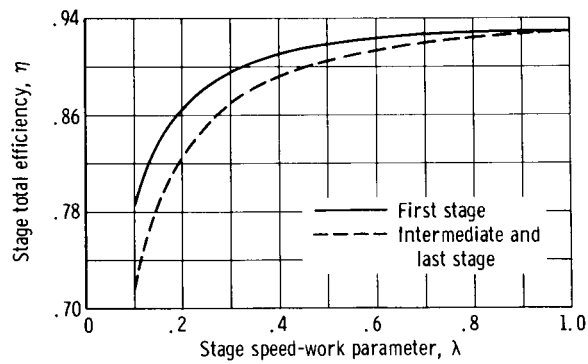


Figure 3. - Computed stage total efficiency characteristics. Symmetrical diagrams; stator-exit angle,  $55^\circ$ ; loss coefficient term, 0.01774.

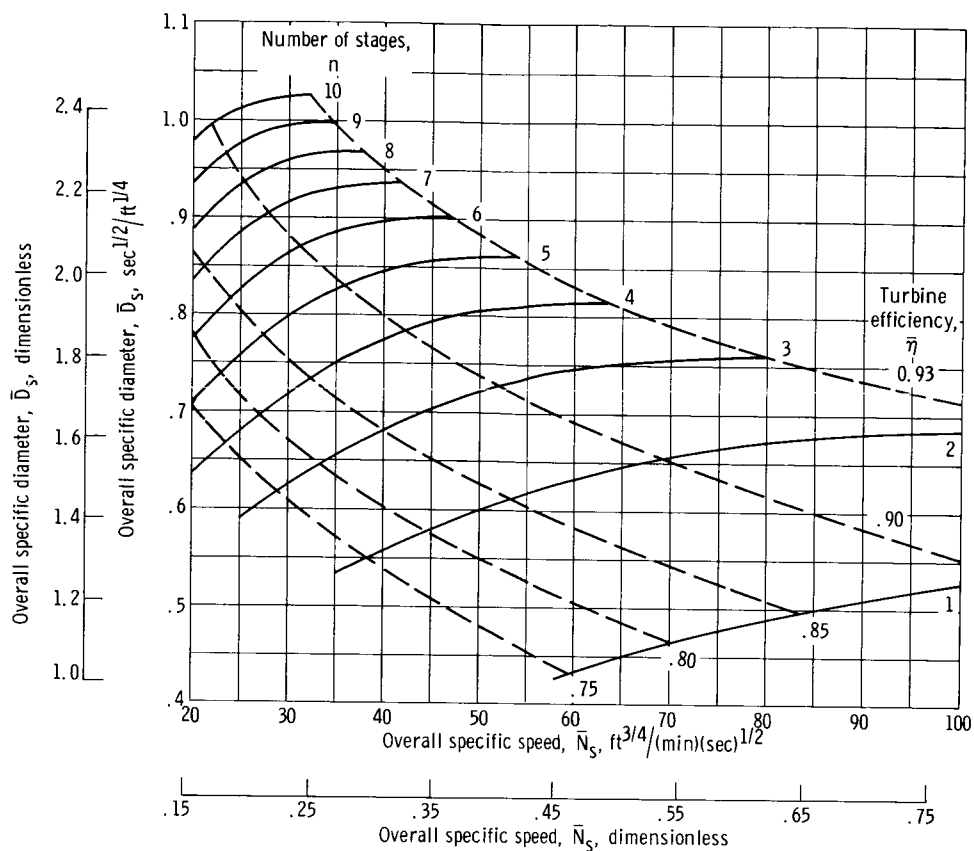


Figure 4. - Generalized turbine geometry-efficiency characteristics. Stator exit angle,  $55^\circ$ ; exit radius ratio, 0.6.

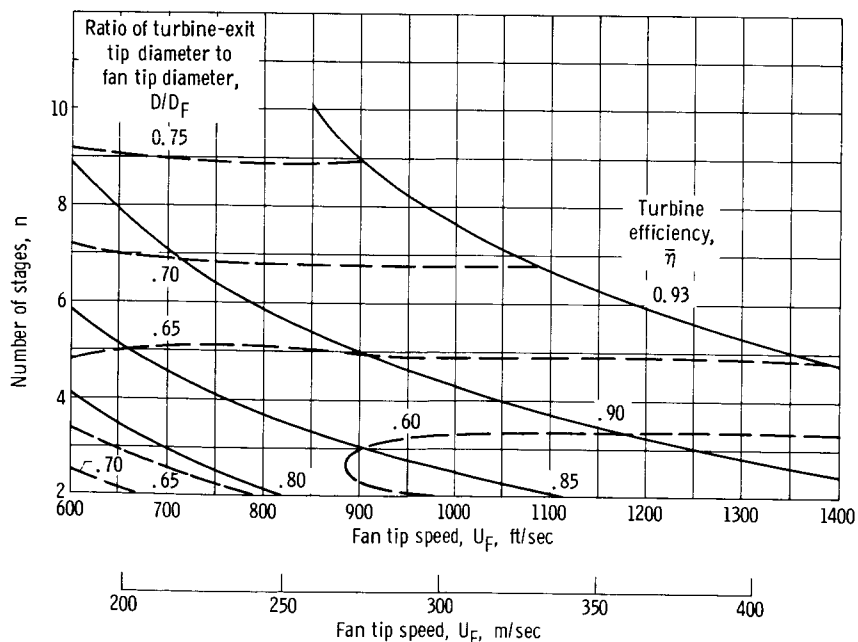


Figure 5. - Effect of fan tip speed on fan-drive turbine geometry-efficiency characteristics. Stator exit angle,  $55^\circ$ ; exit radius ratio, 0.60.

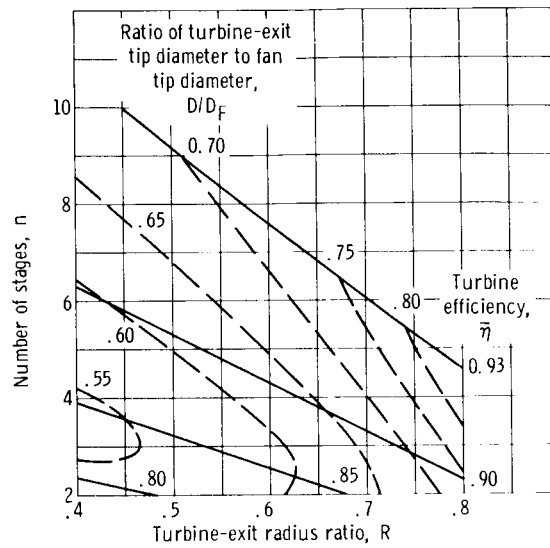


Figure 6. - Effect of turbine-exit radius ratio on fan-drive turbine characteristics. Stator exit angle,  $55^\circ$ ; fan tip speed, 1000 feet per second (304.8 m/sec).

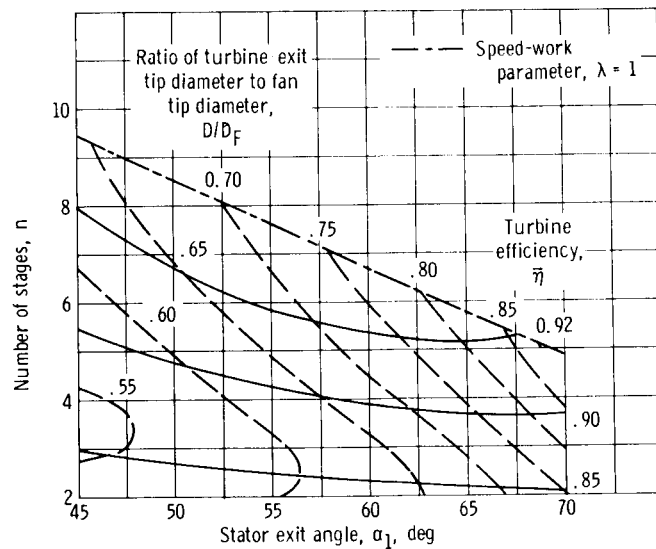


Figure 7. - Effect of stator-exit angle on fan-drive turbine characteristics. Turbine-exit radius ratio, 0.6; fan tip speed, 1000 feet per second (304.8 m/sec).

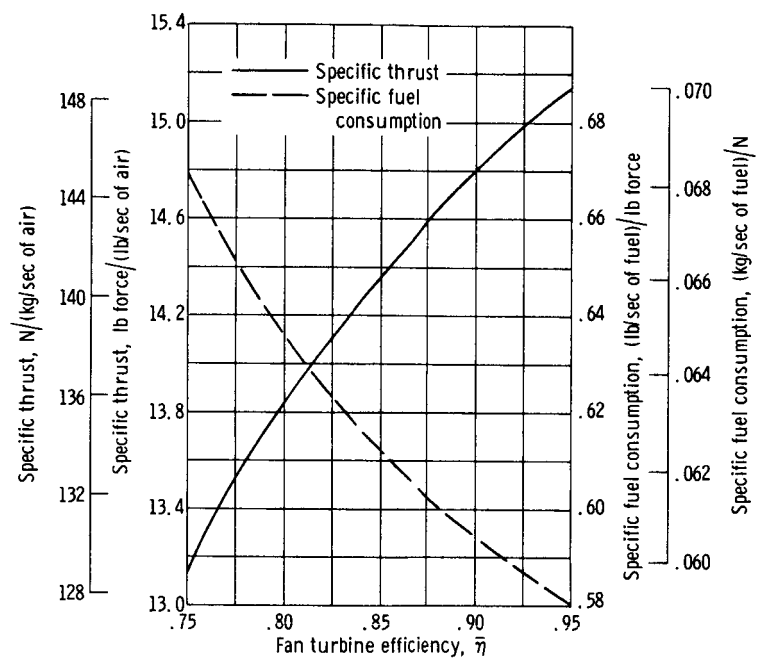


Figure 8. - Effect of fan-turbine efficiency on engine cycle performance.